

## **Igor Herbut: A Modern Approach to Critical Phenomena** **Cambridge University Press, 2007**

**Rudi Podgornik**

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This relatively short and extremely well written book is what I would recommend to every graduate student in condensed matter physics as compulsory reading. I really would. Considering its modest size if compared to other ponderous treatises on the same topic(s), its very clear and well chiseled out explanations and derivations, as well as the care prof. Herbut took in order not to overpower the reader with too extravagant excursions into the mathematical arcana of the field theory, it simply deserves to be noted and read. I am sure the students will appreciate its manageable size and conciseness, its crisp argumentation as well as its elegant and not overwhelming mathematical proofs. There are of course many books on critical phenomena but this one nevertheless stands out. While reading it I wished this would be my first encounter with the methods of field theory applied to condensed matter physics.

The book grew out from the lectures on the theory of phase transitions and the renormalization group that prof. Herbut taught to the grad students at Simon Fraser University. It was conceived primarily not only for the student that wants to get familiar with the notions of phase transitions and critical phenomena, but above all, for the students that actually want to learn how to use and implement the basic concepts of modern theory of critical phenomena, in order to explain and compute the problems s/he is working on. The book thus inevitably contains enough calculational details either explained in the main text or interspersed throughout carefully chosen problems, in order to convey the usefulness and elegance of the field-theoretical techniques.

The book is organized around the principal theme, which was chosen as the superfluid phase transition: it allows a straightforward introduction of the Ginzburg-Landau-Wilson functional, leads naturally to the notion of topological defects and is well supported by the available experimental evidence. Formally the momentum-shell renormalization group approach is given total priority over the real-space renormalization since it flows most naturally from the Ginzburg-Landau-Wilson functional. Besides the introductory chapters, the book

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R. Podgornik (✉)  
NICHD, NIH, 9000 Rockville Pk., Bethesda, 20892-0924, USA  
e-mail: podgornr@mail.nih.gov

contains also several chapters on a more advanced level, nevertheless giving the reader sufficient support in terms of solved problems interspersed throughout the book. Meticulously chosen and worked out in order to illuminate more formal derivations and explanations, the problems fully complement the main text. A short list of fundamental references for each of the chapters is supplied at the end of the book. Together with the problems and selected references this book could thus support a full one semester course on the field-theoretical techniques in condensed matter physics.

As already stated, the eight chapters of the book are extremely concisely written and clearly argued. There is no inessential mathematical clutter and overall the book thus stimulates a very focused and intense reading. The student would need only a basic knowledge of quantum and statistical mechanics in order to follow the subject matter of most of the chapters, everything else is supplied by the textbook itself. The author has obviously taken great care in order to write a self contained and evenly paced text. Chapters 4 and 7 stand out a bit from the main flow of the book since they deal with more advanced material than the rest of the book. Chapter 4 delves into the details of the coupling of the order parameters to other soft modes in superconductors, while Chapter 7 deals with the modern duality transformations. As already noticed by the author, one could omit these two chapters from a more introductory course that the rest of the book would support quite nicely.

The introductory chapter contains basic material on phase transitions and order parameters, the Ising, XY and Heisenberg models serving as illustrations. Universality and scaling of the free energy and correlations serve in order to introduce the notion of critical exponents and the connections between them. Fast paced but clearly explained and illustrated by a wealth of worked out examples.

The second chapter explores the details of the Ginzburg-Landau-Wilson theory for a system of interacting bosons via the coherent state representation and the functional integral formalism. Bose-Einstein condensation for non-interacting bosons is then introduced in order to illuminate the workings of the general formalism that is then extended to a delta-like two body interaction system solved on the Hartree level. The Landau mean-field approximation is introduced next within the framework of the saddle-point approximation of the corresponding Ginzburg-Landau-Wilson functional, leading to the equilibrium mean-field free energy and the scaling function for the field correlations. The chapter concludes with the introduction of the upper critical dimension that separates systems with mean-field like behavior from the others.

The renormalization group transformation is the subject matter of the next chapter. It is introduced within Wilson's momentum-shell transformation. The second order coupling constant or equivalently the chemical potential is then evaluated explicitly to the lowest order in the strength of the interactions with a detailed discussion of its possible behaviors. The fixed point is then introduced based on the emerging behavior of the chemical potential under the RG transformation. The epsilon (4-D) expansion and corrections to scaling are furthermore discussed, leading to the final formulae for the critical exponents  $\nu$  and  $\gamma$  as functions of the number of components of the field, to the second order in epsilon.

Chapter 5 introduces the Goldstone modes and the Mermin-Wagner-Hohenberg theorem on the impossibility of true long-range order at any finite temperature in a 2D system with continuous symmetry. The non-linear sigma model and its low temperature expansion follow next. Chapter 6 elaborates on the Kosterlitz-Thouless transition for vortices and spin waves. The mean-field theory is considered first together with its duality transformation that leads to the sine-Gordon model. The RG transformation of the sine-Gordon model and the Kosterlitz-Thouless transition is then discussed together with the merging of the Kosterlitz-Thouless vortex unbinding critical point with the non-interacting (dual) sine-Gordon formulation. The Heisenberg model in 2D is then analysed, to show its distinction with respect

to the XY model in the sense that it does not lead to an equivalent Kosterlitz-Thouless transition.

The last chapter delves into the theory of quantum phase transitions. They correspond to the non-analyticity of the quantum ground state energy at zero temperature. These phase transitions can be tuned through variations of a coupling constant in the corresponding Hamiltonian, temperature being fixed at zero. The quantum critical point of the  $\psi^4$  theory and the Bose-Hubbard model are treated as examples of this type of phase transitions. The last chapter concludes with several appendices that give some pertinent technical details on the Hubbard-Stratonovich transformation, the linked-cluster theorem and gauge fixing for long-range order. The book closes with a list of relevant but not exhaustive references for further exploration.

This is a well written and clearly argued book. It should be accessible to grad students possibly as their first but maybe even as their only encounter with the field theoretical methods in condensed matter physics. There are no doubt many more detailed and encyclopedic treatises on critical phenomena, but this one stands out by its conciseness, wealth of interesting worked out problems and examples, as well as reader-friendly exposition and self-containedness.